6. D. K. Edwards and W. A. Menard, Comparison of models for correlation of total band absorption, *Appl. Optics* **3**, 621 (1964).
7. C. L. Tien and J. E. Lowder, A correlation for total band absorptance of radiating gases, *Int. J. Heat Mass Transfer* **9**, 689 (1966).
8. Tables of thermal properties of gases, NBS Circular 565 (1955).
9. W. E. Ibele and T. F. Irvine, Jr., Certain Thermodynamics and Transport Properties of the Atmospheres of Venus, Mars and Jupiter, *J. Heat Transfer* **82**, 381 (1960).
10. J. L. Novotny and T. F. Irvine, Jr., Thermal conductivity and Prandtl number of carbon dioxide and carbon dioxide–air mixtures at one atmosphere, *J. Heat Transfer* **83**, 125 (1961).
11. C. L. Tien, Thermal radiation properties of gases, *Adv. Heat Transfer*, **5**, 252 (1968).

Int. J. Heat Mass Transfer. Vol. 11, pp. 1826–1830. Pergamon Press 1968. Printed in Great Britain

DIRECT CONTACT HEAT TRANSFER BETWEEN TWO IMMISCIBLE PHASES DURING DROP FORMATION

C. V. KALYANASUNDARAM, RAJINDER KUMAR and N. R. KULOOR
Department of Chemical Engineering, Indian Institute of Science, Bangalore, India

(*Received* 22 *November* 1967 *and in revised form* 26 *July* 1968)

NOMENCLATURE

A,	surface area [cm^2];
C',	constant;
C_p,	specific heat [cal/g °C];
C_{pc},	specific heat of the continuous phase [cal./g °C];
C_{pd},	specific heat of the dispersed phase [cal./g °C];
H,	total heat content of the drop at any instant [cal];
M,	mass of the drop at any instant, [g]; virtual mass of the drop [g];
P,	constant;
Q,	volumetric rate of flow of the disperesed phase [cm^3/s];
R, R',	constants;
R_c,	capillary radius [cm];
S, S',	constants;
T,	temperature; difference (ΔT) between the temperature of the (internally mixed) forming drop and the temperature of the continuous phase at any instant [°C];
T_1,	initial temperature difference ($T_i - T_c$) [deg C];
V,	drop volume [cm^3];
a, b, c,	coefficients in the polynomials;
d,	drop diameter [cm];
g,	acceleration due to gravity [cm/s^2];
h,	heat-transfer coefficient [cal./s cm^2 °C];
k,	thermal conductivity [cal. cm/s cm^2 °C];
m,	mass rate of entry of the dispersed phase [g/s];
r,	drop radius [cm];
t,	time [s];
v_e,	velocity of expansion (of the drop) in the first stage of formation [cm/s];
v_r,	velocity of rise (of the drop) in the detachment period [cm/s];
x, y,	coordinates.

Greek symbols

ρ,	density [g/cm^3];
$\Delta\rho$,	density difference ($\rho_c - \rho_d$) [g/cm^3];
μ,	viscosity [P, c P];
v,	interfacial tension [dyn/cm].

Subscripts

s,	of static drop;
c,	continuous phase;
d,	dispersed phase;
e,	expansion stage;
f,	formation stage;
i,	inlet drop;
o,	outlet drop;
r,	of rising drop.

Dimensionless groups

N_{Nu},	nusselt number (hd/k);
N_{Re},	reynolds number ($dv\rho/\mu$);
N_{Pr},	prandtl number ($C_p\mu/k$).

Investigations on mass transfer in systems involving a fluid sphere and a liquid continuous medium have normally indicated a considerably large proportion of transfer in the region of formation. The observations on mass transfer can be considered to hold good, in general, for heat transfer also.

Some of the earlier workers [1, 2] in this field have attempted to explain the proportions of transfer in the formation zone on the basis of Higbie's penetration theory [3]. Most of the others [4–7] have confined themselves to the experimental estimation of the proportion transferred in the end regions by extending the usually linear plot of total transfer vs. column height to zero column height.

More recently, Sideman and coworkers have extensively investigated [8–11] the mechanism of evaporation of a volatile drop moving in an immiscible liquid medium. These investigations, relevant to desalination projects, have been aimed primarily at the free ascent region.

The investigations reported here aim at presenting data on heat transfer from forming drops and correlating the data on the basis of an assumed mechanism of drop formation and heat transfer.

The heat-transfer studies have been carried out in the absence of mass transfer between the two phases.

EXPERIMENTAL

The experimental set-up represented in Fig. 1 consists of a constant pressure air supply, an arrangement for drying the pressurized air, a dispersion section, an arrangement for creating the desired temperature gradient between the dispersed and continuous phases, thermocouples for the measurement of the temperatures of the two phases during a "run", the eductor tube containing the continuous phase and finally, the drop collection and measurement units.

The hot drops, formed at the capillary (16), get detached, follow a free ascent and coalesce at the interface (21) which extends almost into the stem of the inverted funnel (18).

The hot junction of the outlet probe (22) is located as close to the interface as possible; as each drop reaches the interface, the thermocouple records just the temperature of the coalescing drop.

For a particular set of drop size, time of formation and inlet drop temperature, the outlet temperature of the drop is measured at various times of contact allowed for transfer between the two phases in the free ascent period.

Further, extra experiments were conducted under conditions identical to the ones employed earlier in the regular "runs", for quantitative estimation of the fall in temperature of the drop forming liquid as it travels from the point in T piece (12) (at which the inlet temperature of the drop was earlier measured) to the tip of the capillary.

For the purpose of calculation, the inlet temperature of the drop forming liquid, measured below the tip of the capillary just prior to drop formation in these experiments, has been used.

Data were collected dispersing hot organic phases such as benzene, xylene and toluene in a heavier continuous medium (distilled water).

The drop size has been varied by varying the diameter of the drop forming capillary (0·1150–0·4600 cm).

The physical properties of the systems studied have been taken from literature [12–15].

RESULTS AND DISCUSSION

To obtain the value of T_0 (the temperature of the drop just at completion of its formation), the experimental values of the temperature of the drop at different times of contact in the free ascent period, are extrapolated to zero time of contact.

An equation of the form

$$y = a + bx + cx^2 \qquad (1)$$

has been used for extrapolation. The value of T_0 is obtained by putting $x = 0$ in the above equation. Direct measurements of the temperature of the drop at the end of formation are compared in Fig. 2 with the corresponding extrapolated values of T_0 (in terms of fall in temperature of the drop during formation).

The comparison, made under typical operating conditions, is satisfactory.

QUANTITATIVE EVALUATION OF RESULTS

Quantitative prediction of results is attempted here assuming the presence of a boundary layer around the forming drop which in turn is assumed to be internally agitated. The dynamics of the drop as represented by the two-step mechanism of formation [16, 17] has been considered for the evaluation of Reynolds number on which the characteristics of the boundary layer will depend.

The drop is further assumed to be spherical and to start growing from the tip of the capillary. Further assumptions regarding constancy of the temperature of the continuous medium and its physical properties are also made.

QUANTITATIVE DISCUSSION

The heat balance equation in the present case is of the form

$$(mC_{pd}T_1) - (hAT) = MC_{pd}\frac{dT}{dt} + C_{pd}T\frac{dM}{dt}. \qquad (2)$$

The evaluation of the heat-transfer coefficient (denoted by h) is made on the basis of the following boundary layer equation [18, 19].

$$N_{Nu} = C'(N_{Re})^{0\cdot50}(N_{Pr})^{0\cdot33}. \qquad (3)$$

In equation (3), C' is a constant the value of which has been taken as 0·70 on the basis of the assumption that nearly 50 per cent of the transfer occurs in the wake region behind the forming drop [20].

In the first stage (termed the expansion stage) of its formation, the drop expands with a certin velocity v_e which can be expressed by dr/dt, the rate at which the radius of the drop increases; i.e.

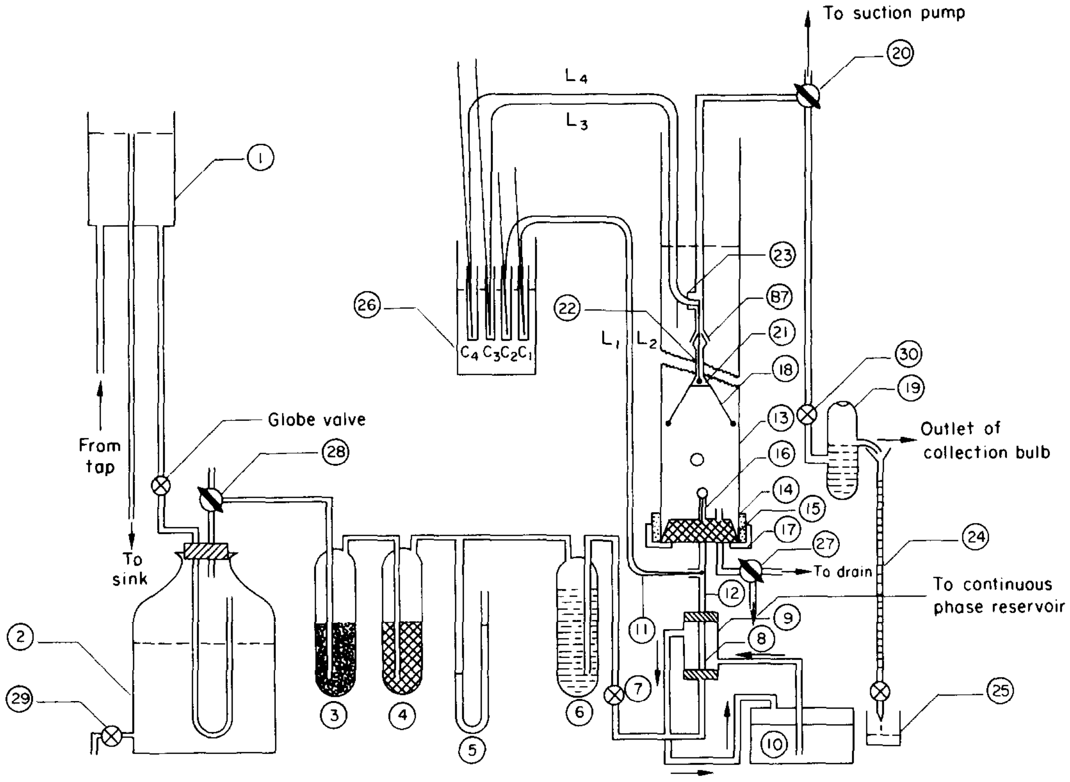

FIG. 1. Flow diagram of the experimental set-up for studies in heat transfer in drops.

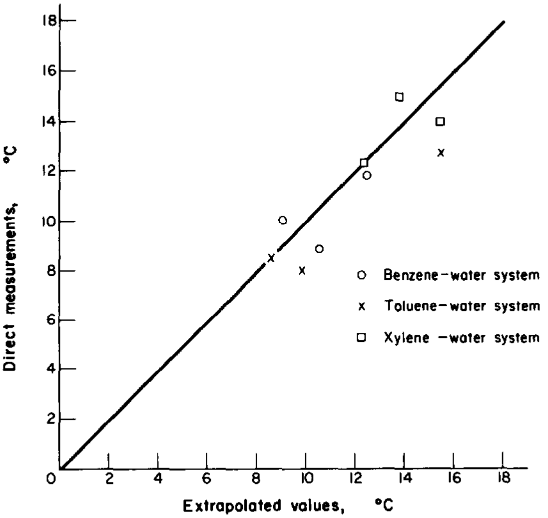

FIG. 2. Fall in temperature of drop during formation: comparison of extrapolation with direct measurements.

$$v_e = \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{Q}{4\pi r^2}. \tag{4}$$

In the second step (termed the detachment stage), the drop is assumed to accelerate from the tip. The velocity v_r with which the drop rises in the detachment step is given by

$$v_r = \left[\frac{Q \cdot \Delta\rho \cdot g}{6\pi\, r_s\, \mu_c}\right] t. \tag{5}$$

The velocity factors shown in equations (4) and (5) are introduced in the boundary layer equation (3) for the evaluation of the corresponding Reynolds number (and subsequently the heat-transfer coefficient) for the two stages of formation.

The mass, surface area, diameter and radius of the drop are also expressed as functions of time in the analysis.

The heat balance equation for the first stage of drop formation takes up the form,

$$\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) = \frac{1}{Rt}(P - RT - St^{\frac{1}{6}}T). \tag{6}$$

And, for the detachment period, the equation is of the form,

$$\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) = \left(\frac{1}{R' + Rt}\right)(P - RT - S't^{\frac{1}{2}}T). \qquad (7)$$

In equations (6) and (7),

$$P = mC_{pd}T, R = mC_{pd}, R' = m_sC_{pd}$$

$$S = C'k_c\left[\frac{\rho_c}{6\mu_c}\right]^{0\cdot50}\left[\frac{C_{pc}\mu_c}{k_c}\right]^{0\cdot33}(4\pi)^{\frac{1}{3}}(3Q)^{\frac{2}{3}}$$

and

$$S' = C'k_c\left[\frac{Q\,.\,\Delta\rho\,.\,g\,.\,\rho_c}{12v_s^2\mu_c^2}\right]^{0\cdot50}\left[\frac{C_{pc}\mu_c}{k_c}\right]^{0\cdot33}.$$

Equations (6) and (7) respectively represent the variation of T (the difference between the temperature of the forming drop and the temperature of the continuous medium at any instant) with time in the two distinct stages of formation [16, 17] of the drop.

These non-linear differential equations have been solved by the method of Runge and Kutta [21],

The value of T at the end of time t_e—the period of expansion given by V_s/Q—obtained by solution of equation (6), is taken as the initial value of T for the second stage equation (7). Harkin's correction [22] has been applied for the evaluation of V_s, the static drop volume, corresponding to the first stage of drop formation [16, 17].

The solution of equation (7) at the end of time t_r—the period of drop detachment defined by $(t_f - t_e)$—yields the final value of T at the end of formation of the final drop.

The actual temperature of the drop at the end of formation of the total drop can therefore be evaluated on the basis of the present model as T represents precisely the difference between the temperature of the forming drop and that of the continuous phase at any instant.

VERIFICATION OF THE PROPOSED MECHANISM

The experimental values of the fall in temperature of the drop during formation and the ones predicted by the proposed model are compared in Table 1. The agreement is good, though the experimental values are generally higher; this may be explained as due to the occurrence of a finite amount of transfer in the coalescence region and also due to the assumption of constant area of the drop in the detachment stage. The deviations, however, are quite small (of the order of 10 per cent).

REFERENCES

1. P. M. Heertjes, W. A. Holve and H. Talsma, Mass transfer between isobutanol and water in a spray-column, *Chem. Engng Sci.* **3**, 122–142 (1954).
2. W. Licht, Jr. and W. F. Pansing, Solute transfer from single drops in liquid–liquid extraction, *Ind. Engng Chem.* **45**, 1885–1896 (1953).
3. R. Higbie, The rate of absorption of a pure gas into a still liquid during short periods of exposure, *Trans. Am. Inst. Chem. Engrs* **31**, 365–389 (1953).
4. A. I. Johnson and A. E. Hamielec, Mass transfer inside drops, *A.I.Ch.E.Jl* **6**, 145–149 (1960).
5. T. K. Sherwood, J. E. Evans and J. V. A. Longcor, Extraction in spray and packed columns, *Ind. Engng Chem.* **31**, 1144–1150 (1939).
6. F. B. West, A. J. Herrman, A. T. Chong and L. E. K. Thomas, Addition agents and interfacial barriers in liquid–liquid extraction, *Ind. Engng Chem.* **44**, 625–631 (1952).
7. W. Licht, Jr. and J. B. Conway, Mechanism of solute transfer in spray towers, *Ind. Engng Chem.* **42**, 1151–1157 (1950).
8. S. Sideman and Y. Taitel, Direct contact heat transfer with change of phase: evaporation of drops in an immiscible liquid medium, *Int. J. Heat Mass Transfer* **7**, 1273–1289 (1964).
9. S. Sideman and Z. Barsky, Turbulence effect on direct contact heat transfer with change of phase: Effect of mixing on heat transfer between evaporating volatile liquid in direct contact with an immiscible liquid medium, *A.I.Ch.E. Jl* **11**, 539–545 (1965).
10. S. Sideman and G. Hirsch, Direct contact heat transfer with change of phase: Condensation of single vapour bubbles in an immiscible liquid medium: preliminary studies, *A.I.Ch.E.Jl.* **11**, 1019–1025 (1965).
11. S. Sideman and Y. Gat, Direct contact heat transfer with change of phase: spray column studies of a three-phase heat exchanger, *A.I.Ch.E.Jl* **12**, 296–303 (1966).
12. C. D. Hodgman, *Handbook of Chemistry and Physics*, 44th edn, pp. 2258–2263. Chemical Rubber, Cleveland, Ohio (1962–1963).
13. F. H. Newman and V. H. L. Searle, The general properties of matter, 4th edn, p. 182. Edward Arnold, London (1948).
14. J. H. Perry, *Chemical Engineers' Handbook*, 2nd edn, p. 775. McGraw-Hill, New York (1941).
15. T. F. Young and W. D. Harkins, Interfacial tension for solid–liquid and liquid–liquid interfaces, in *International Critical Tables of Numerical Data, Physics, Chemistry and Technology*, edited by E. W. Washburn *et al.*, Vol. 4, p. 436. McGraw-Hill, New York (1928).
16. E. V. L. Narasinga Rao, R. Kumar and N. R. Kuloor, Drop formation studies in liquid–liquid systems, *Chem. Engng Sci.* **21**, 867–880 (1966).
17. C. V. Kalyanasundaram, Heat transfer in dispersed particles, Ph.D. Thesis, Indian Institute of Science, Bangalore, India (1967).
18. F. H. Garner and R. B. Keey, Mass transfer from single solid spheres—1. Transfer at low Reynolds numbers, *Chem. Engng Sci.* **9**, 119–129 (1958).
19. S. K. Friedlander, Mass and heat transfer to single spheres and cylinders at low Reynolds numbers, *A.I.Ch.E.Jl* **3**, 43–48 (1957).
20. P. Harriot, A review of mass transfer to interfaces, *Can. J. Chem. Engng* **40**, 60–69 (1962).
21. J. B. Scarborough, *Numerical Mathematical Analysis*, p. 356. Oxford Book, Calcutta (1964).
22. W. D. Harkins and F. E. Brown, The determination of

surface tension (free-surface energy), and the weight of falling drops: the surface tension of water and benzene by the capillary height method, *J. Am. Chem. Soc.* **41**, 499–524 (1919).

Table 1. Verification of the proposed model: comparison of the experimental and calculated values of the fall in temperature of the drop during formation

System	Capilary diameter (cm) (cm)	Inlet temperature of drop (°C)		Temperature of continuous phase (°C)	Time of formation (s)	Fall in temperature of drop during formation (°C)	
		at *T* piece	near capil. tip.			Experimental	Calculated
Benzene–water	0·1150	59·90	48·50	22·90	2·84	22·93	18·78
	0·1150	57·40	47·60	22·55	1·85	23·51	19·00
	0·1150	54·30	47·50	23·10	1·80	23·43	18·80
	0·1890	58·80	46·00	24·10	2·93	20·09	15·65
	0·1890	53·80	42·50	24·20	2·96	16·25	11·76
	0·2450	54·70	45·00	26·70	2·95	14·80	11·16
	0·2450	49·70	43·00	26·40	3·04	9·12	10·20
	0·3356	62·00	46·50	27·80	2·91	15·73	13·95
	0·3356	58·20	42·50	27·70	2·90	10·64	10·77
	0·4140	61·80	45·50	24·80	3·04	17·68	14·00
	0·4140	57·50	46·00	24·60	3·00	18·30	11·90
	0·4140	53·70	41·50	24·80	2·99	13·92	11·30
	0·4600	58·40	45·00	24·55	2·99	14·75	13·65
	0·4600	56·60	45·00	25·00	2·99	15·18	12·20
	0·4600	52·60	42·00	23·20	3·03	14·60	11·68
Toluene–water	0·1600	48·80	41·50	28·50	2·03	9·89	6·09
	0·1940	53·70	42·50	21·50	3·00	19·38	15·14
	0·1940	48·40	41·50	23·20	2·95	17·23	12·20
	0·2450	61·10	49·50	22·10	3·00	24·63	14·12
	0·2450	50·20	40·50	23·30	3·05	15·50	12·55
	0·3130	54·50	48·80	27·50	2·53	19·68	14·20
	0·3130	51·00	41·50	28·25	2·76	8·62	8·35
	0·3130	69·20	53·00	26·50	2·48	23·37	16·60
	0·3356	67·80	52·50	29·90	3·00	17·66	12·60
	0·3356	47·80	40·50	29·50	2·82	7·24	7·46
Xylene–water	0·3356	59·70	45·50	24·80	3·02	15·40	10·44
	0·3356	49·10	36·00	25·00	3·03	8·50	6·27
	0·4140	62·40	45·30	23·30	3·00	15·43	14·80
	0·4140	55·80	42·00	24·55	3·02	14·66	10·00
	0·4140	46·20	38·50	23·95	3·02	10·78	9·75
	0·4600	64·00	47·50	24·30	3·01	13·70	12·20
	0·4600	52·80	42·50	24·15	3·04	15·38	9·62
	0·4600	47·00	38·00	23·95	3·01	8·40	10·17